Modelling hypoxia in the Hamilton Harbour, Ontario, Canada: A Bayesian approach

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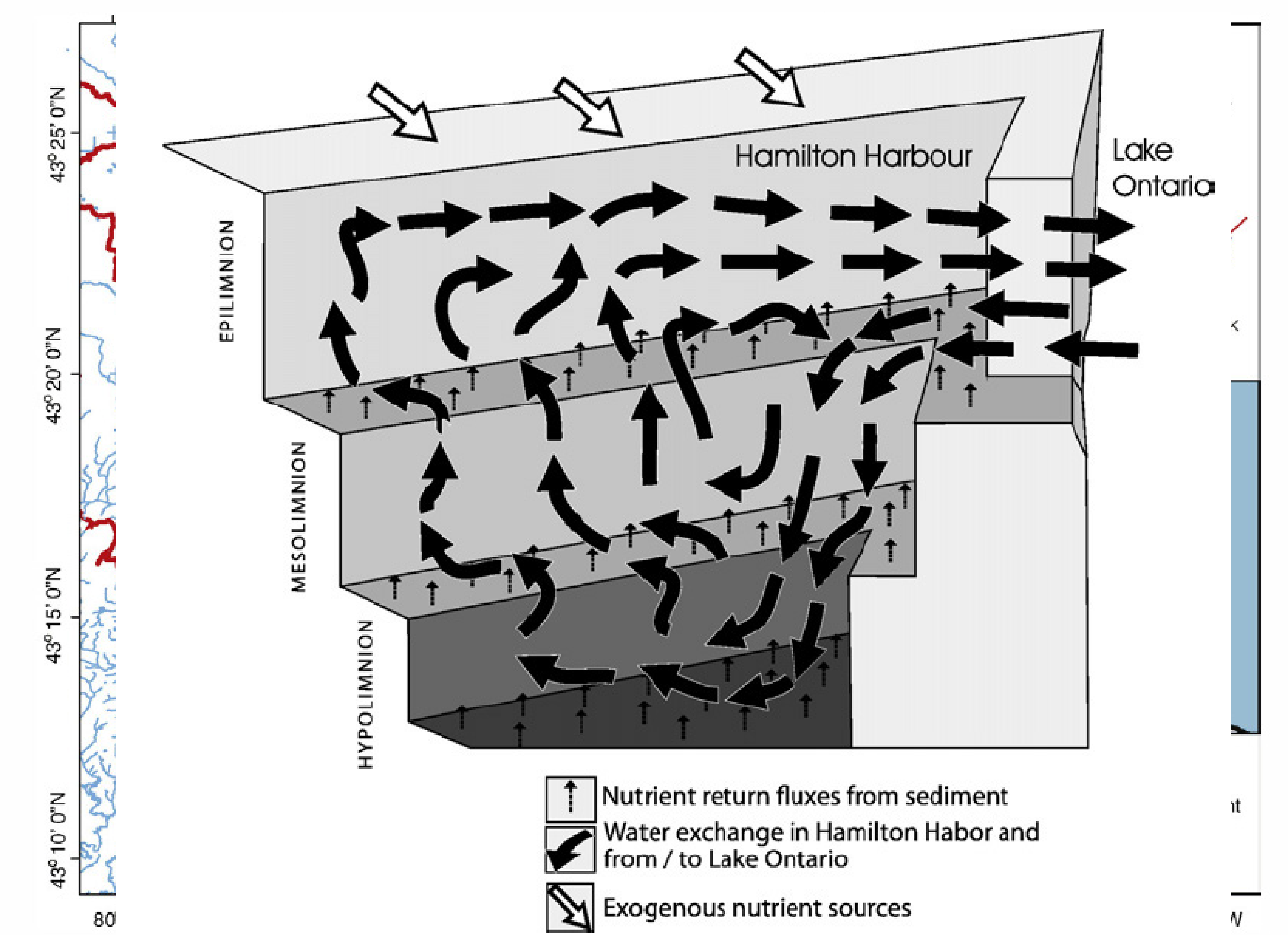

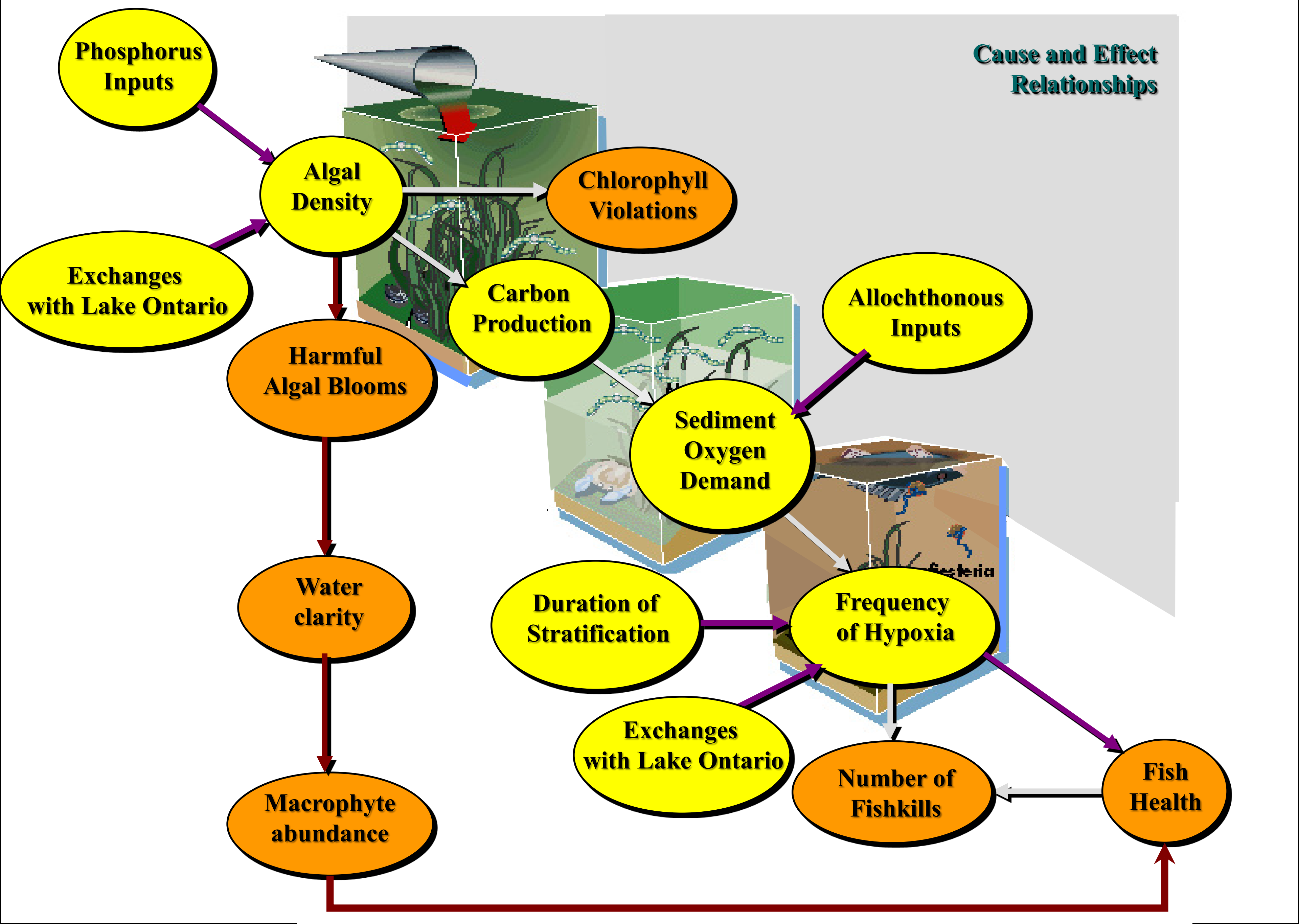

Cause and Effect Relationships
Phosphorus Inputs
Algal Density
Exchanges with Lake Ontario
Chlorophyll Violations
Carbon Production
Harmful Algal Blooms
Allochthonous Inputs
Sediment Oxygen Demand
Water clarity
Duration of Stratification
Frequency of Hypoxia
Exchanges with Lake Ontario
Number of Fishkills
Fish Health
Macrophyte abundance

Water quality standards

TP loading $\leq$ *142 kg day*$^{-1}$

Total Phosphorus $\leq$ *17 (or 20) µg L*$^{-1}$

Chlorophyll a $\leq$ *5-10 µg L*$^{-1}$

Secchi disk depth $\geq$ *3 m*

DO $\geq$ *4 mg L*$^{-1}$

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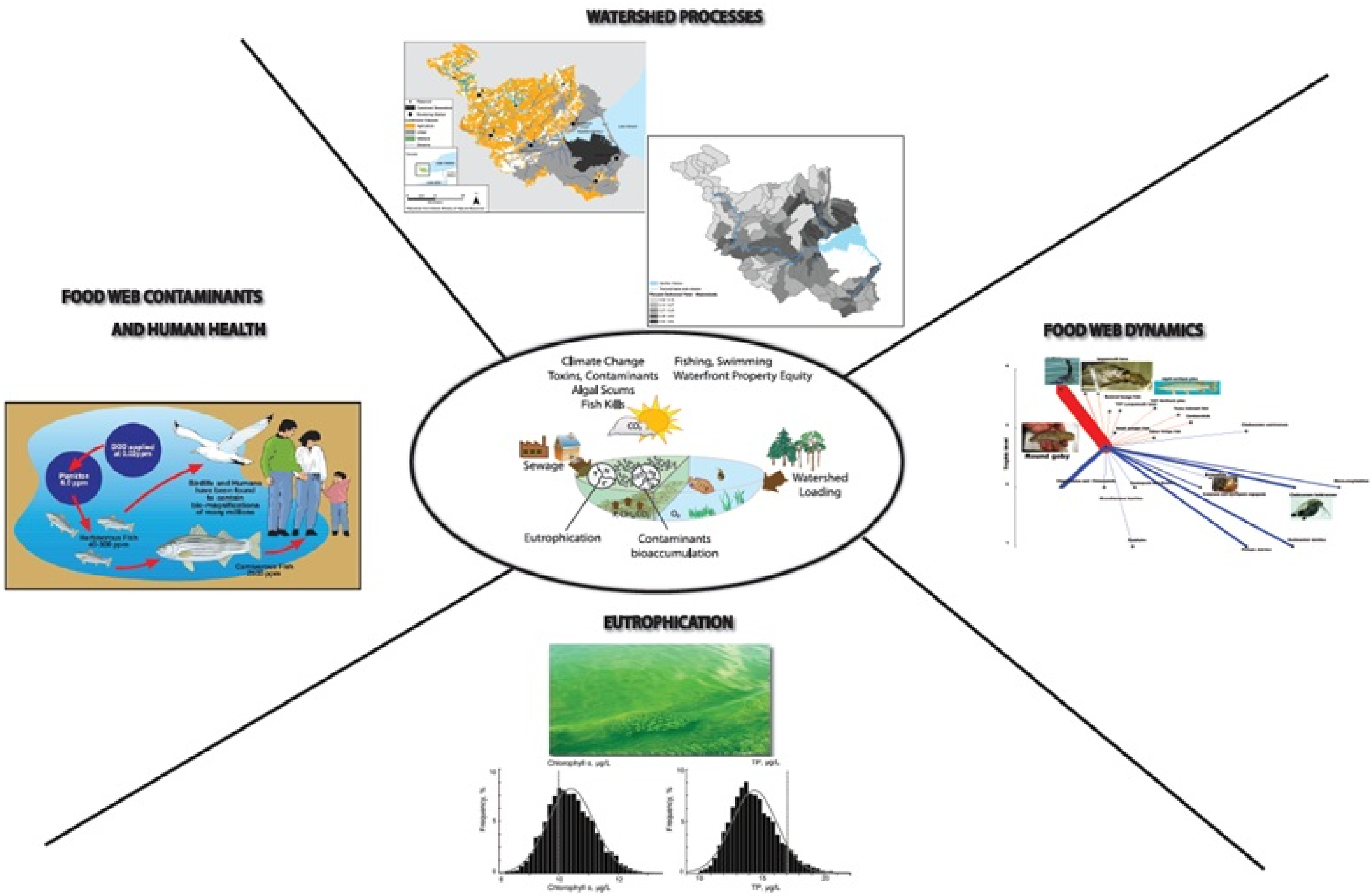
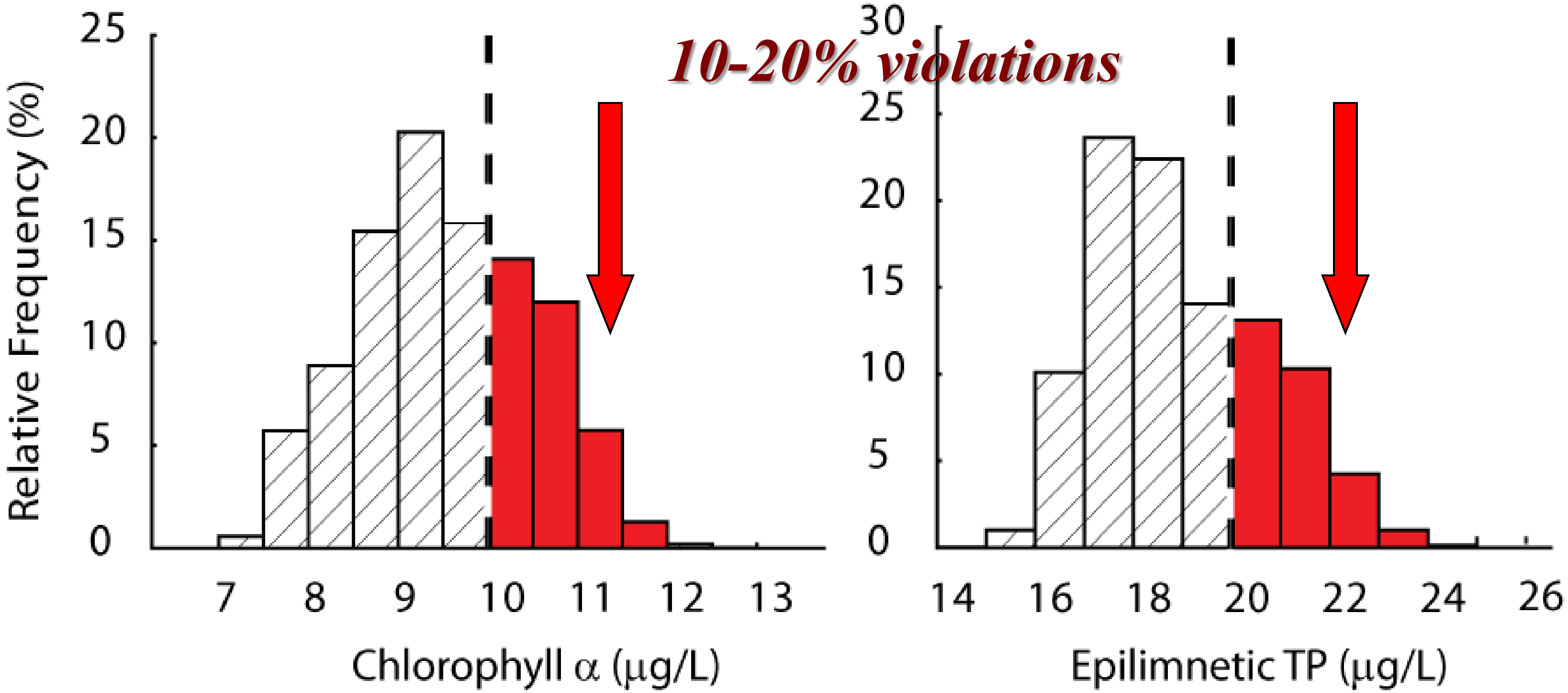


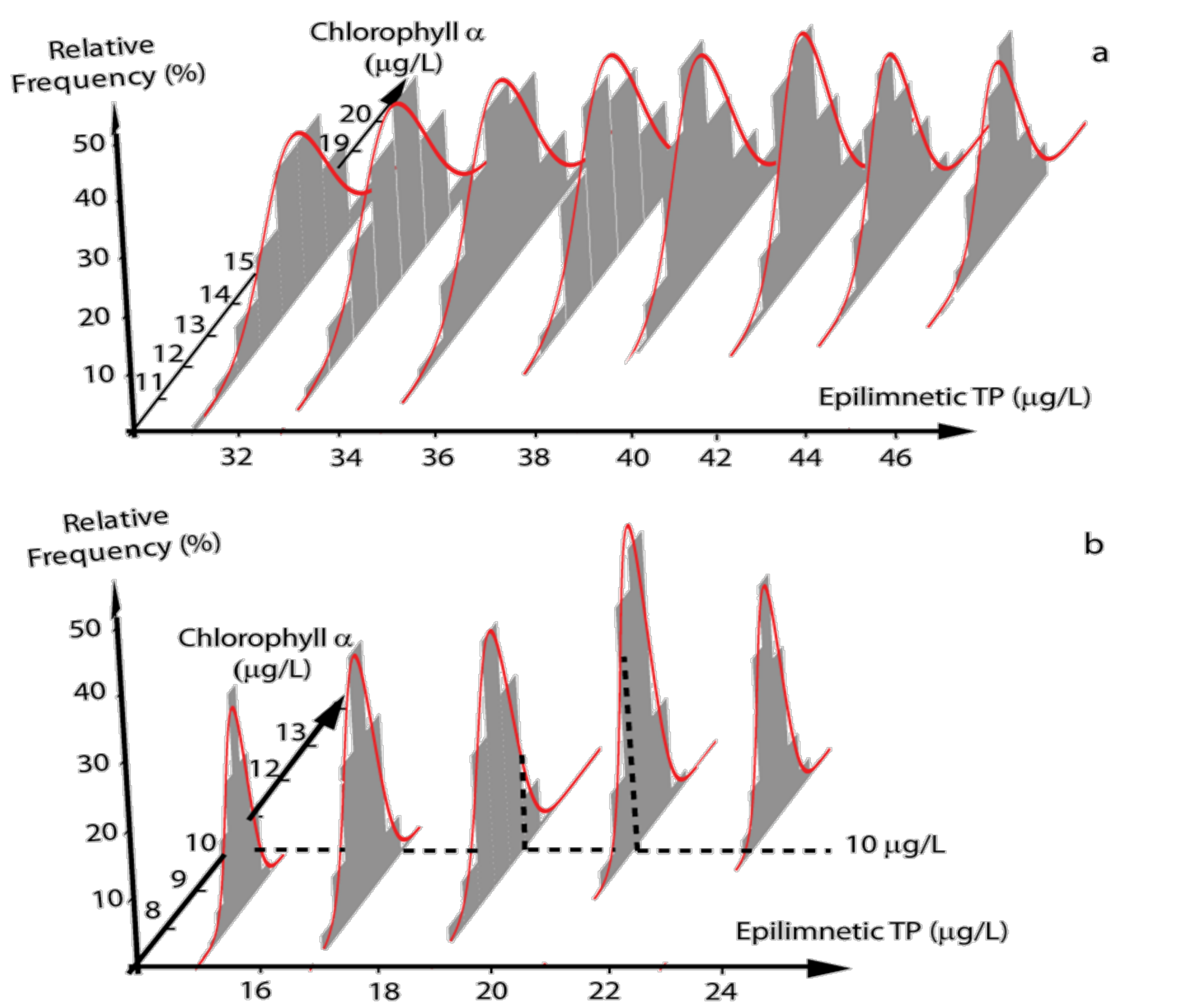
Probabilistic projection of system response to nutrient loading reduction strategies



Chlorophyll a predictive distributions for different levels of Total Phosphorus

Present loading conditions

Nutrient loading reduction



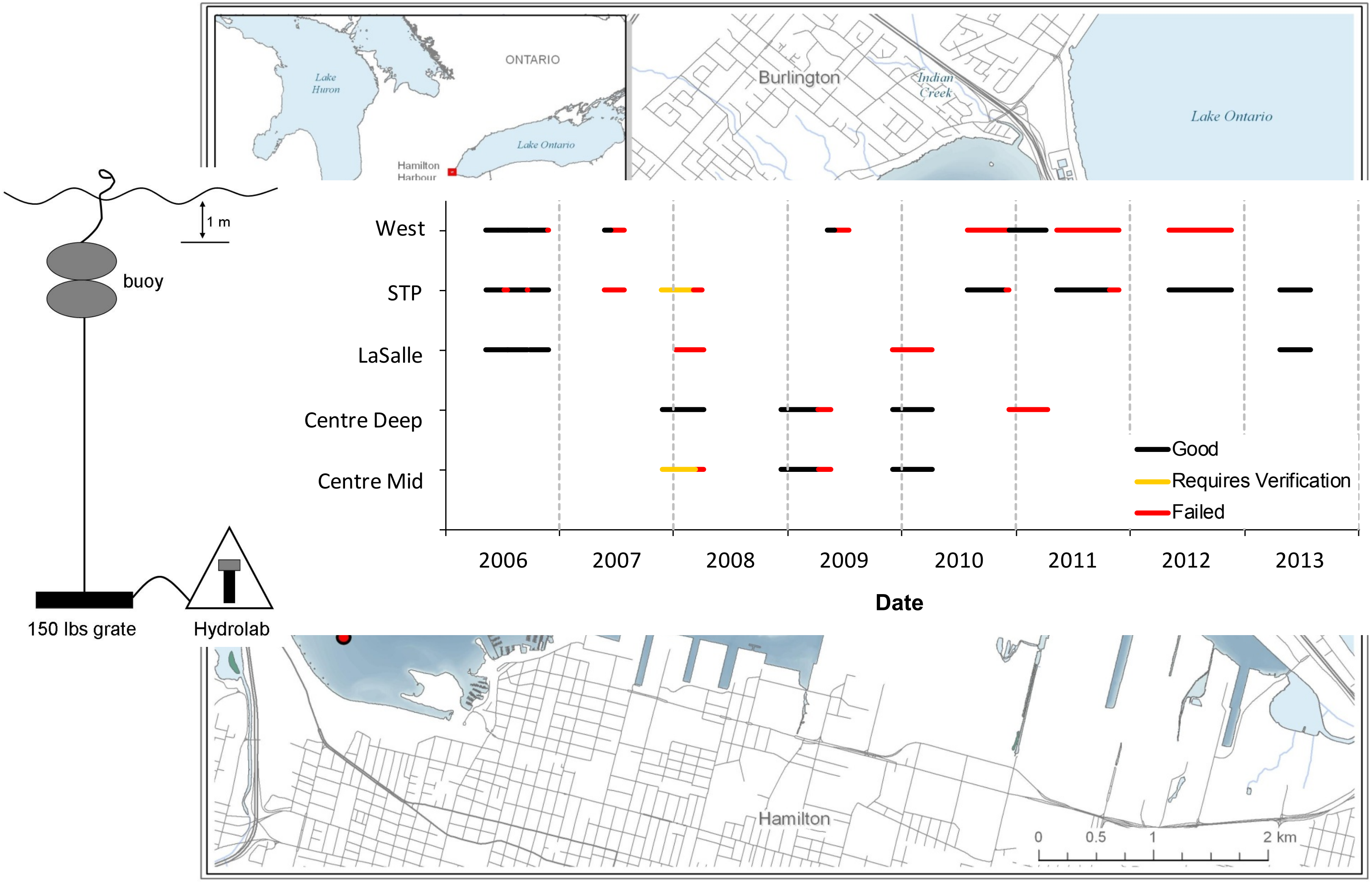
Objectives

• How possible is it to meet the *DO* delisting objective, if the nutrient loading reductions proposed by the Hamilton Harbour Remedial Action Plan are actually implemented?

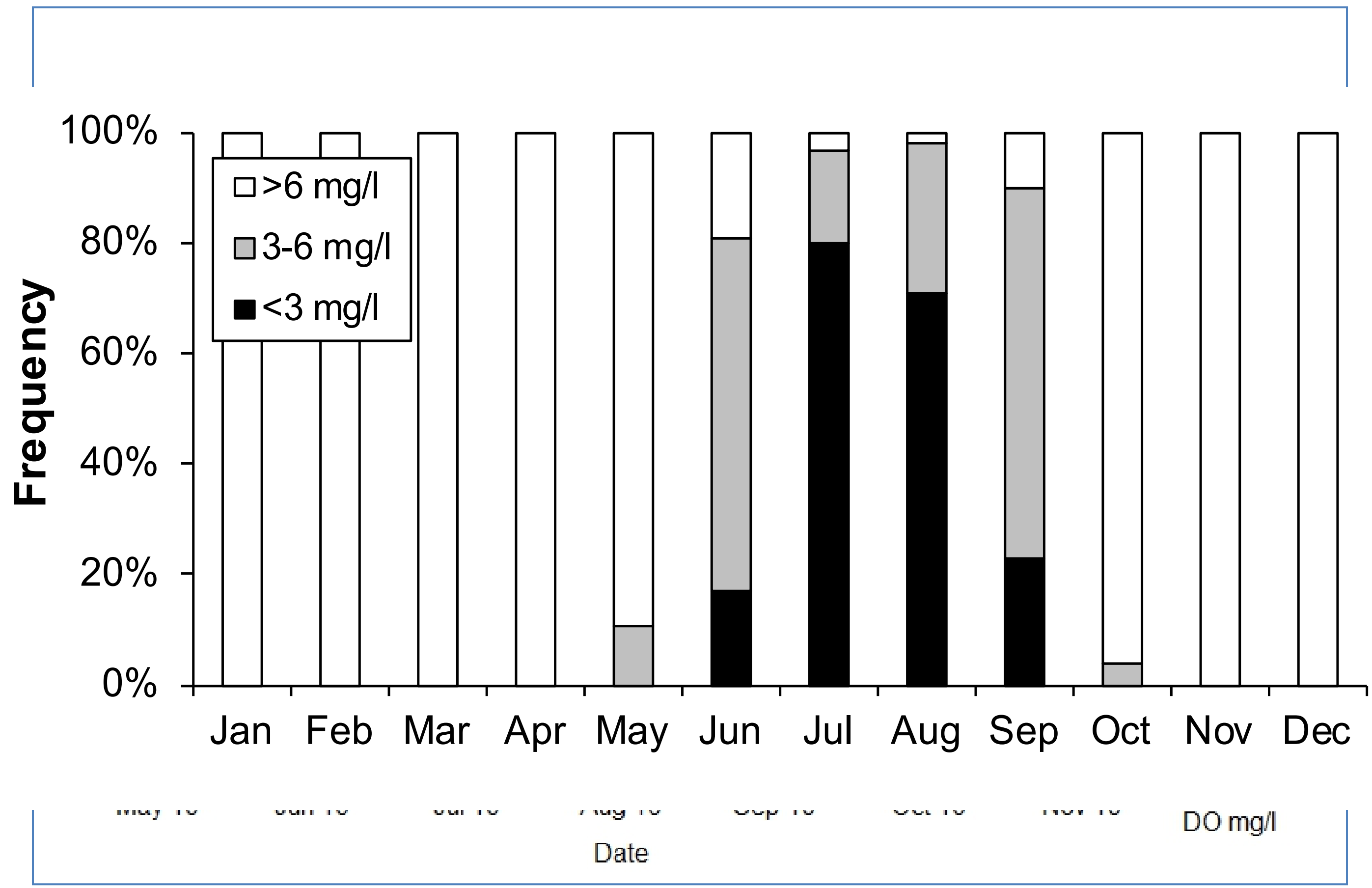
• What additional remedial actions are needed to increase the likelihood of meeting the DO target?

•What are the major sources of uncertainty that will ultimately determine the attainment of the existing delisting goal?

Spatial Segmentation



Spatial Segmentation



Bayesian Approach

In modeling context:

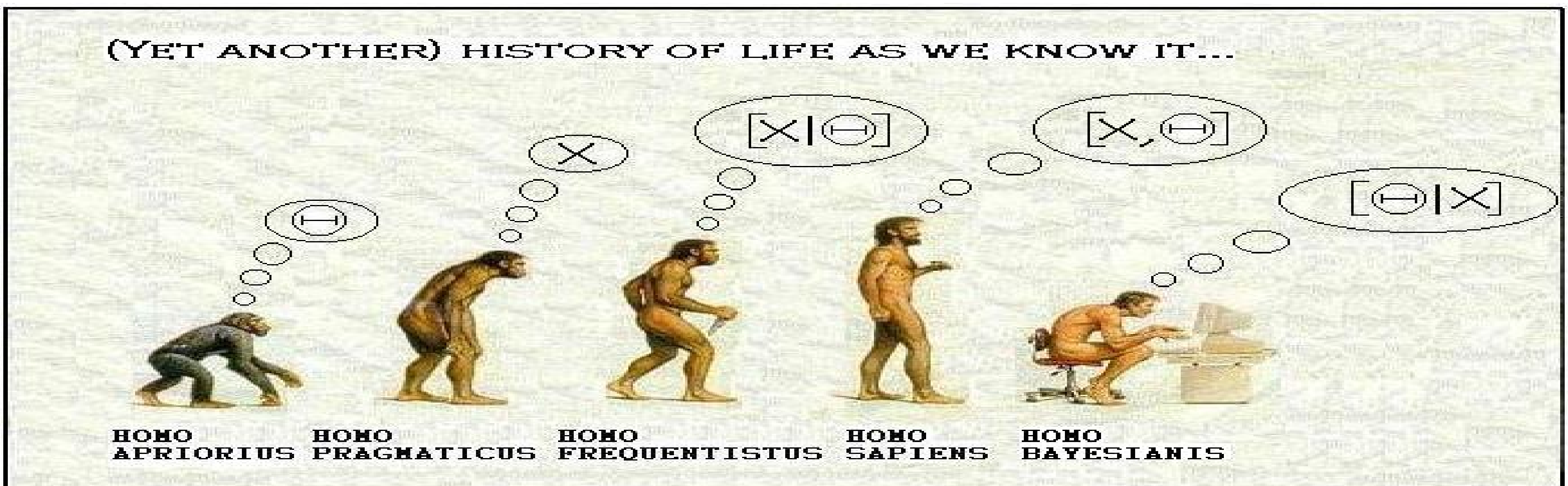
$$P(Model|Data) = \frac{P(Data|Model)\,P(Model)}{P(Data)}$$

$$\propto P(Data|Model)\,P(Model)$$

Future

Present

Past



Bayesian DO modelling

$$DO_{i,t} = \beta_{0i} + \sum_j \beta_j \cdot X_{j,i,t} + \delta_t + \varepsilon_{i,t}$$

$i = 1,\ldots,5 \quad j=1,2 \quad t=1,\ldots,365$

Spatially variant intercept

Causal factors of hypoxia

- **Sediment Oxygen Demand (eutrophication model)**
- **Hydrodynamic patterns (lake thermal stratification)**

Conditional autoregressive term to accommodate the serial correlation of the daily data

Model structural error

Bayesian DO modelling

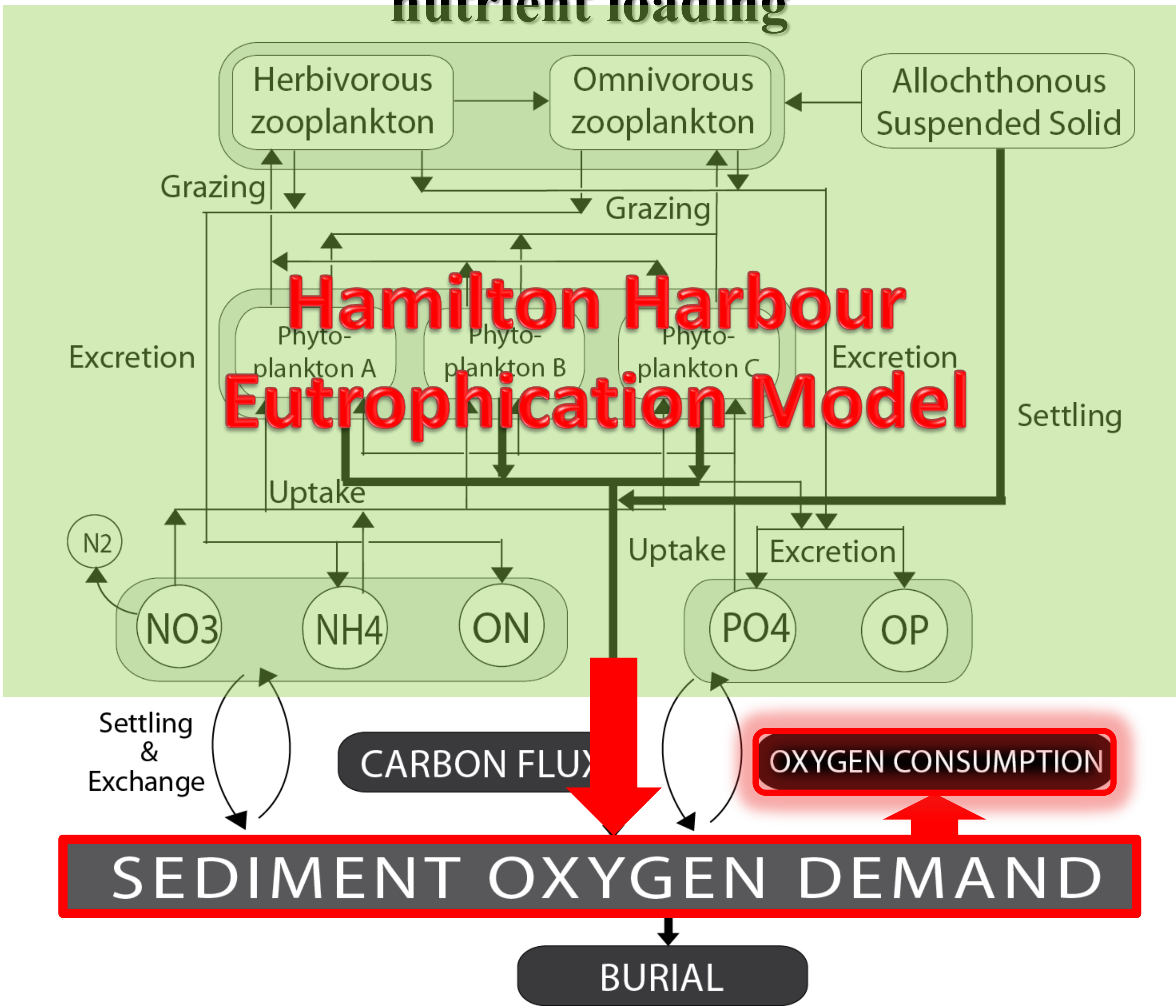
$$N(2\delta_{t+1} - \delta_{t+2}, \omega^2) \qquad \textit{for } t = 1$$

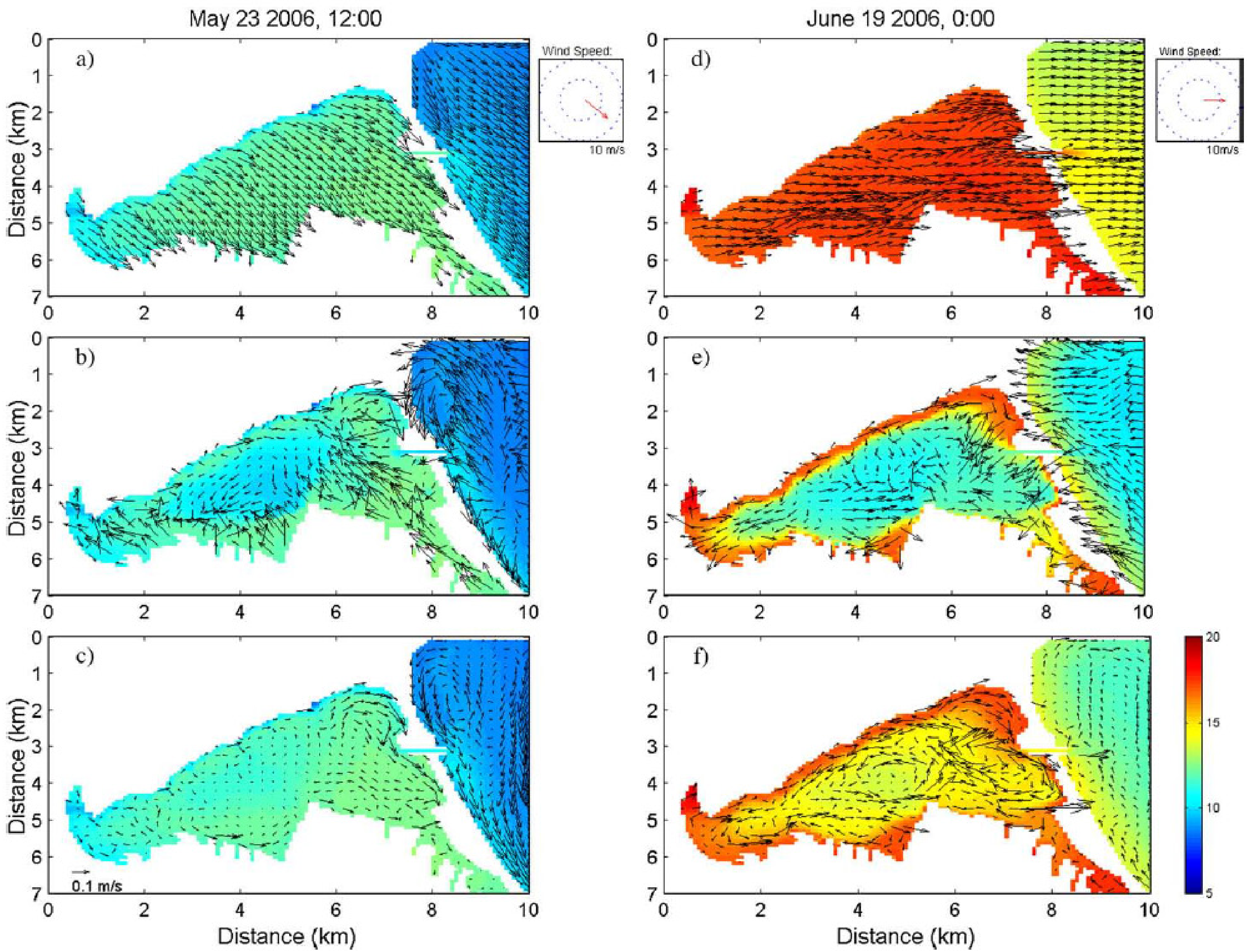
$$P(\ p(\delta_t|\delta_{-t}, \omega^2) \sim \begin{cases} N(\delta_{t+1}, \omega^2) \textit{for } t = 1 \\ N\left(\dfrac{\delta_{t-1} + \delta_{t+1}}{2}, \dfrac{\omega^2}{2}\right) \textit{for } t = 2, \ldots, T-1 \\ N(\delta_{t-1}, \omega^2) \textit{for } t = T \end{cases} \quad \ldots,\ T\text{-}2$$

$$N(\delta_{t-2} - 2\delta_{t-1}, \omega^2) \qquad \textit{for } t = T$$

Conditional autoregressive term to accommodate the serial correlation of the daily data

Device for evaluating SOD response to external nutrient loading





May 23 2006, 12:00
June 19 2006, 0:00
a)
b)
c)
d)
e)
f)
Wind Speed:
10 m/s
10m/s
0.1 m/s
Distance (km)
Distance (km)
20
15
10
5

Bayesian Kriging

multivariate Gaussian distribution with covariance matrix expressed as a parametric function of distance between pairs of points

$$f(d_{xy};\phi,\kappa) = \exp[-(\phi \cdot d_{xy})^{\kappa}]$$

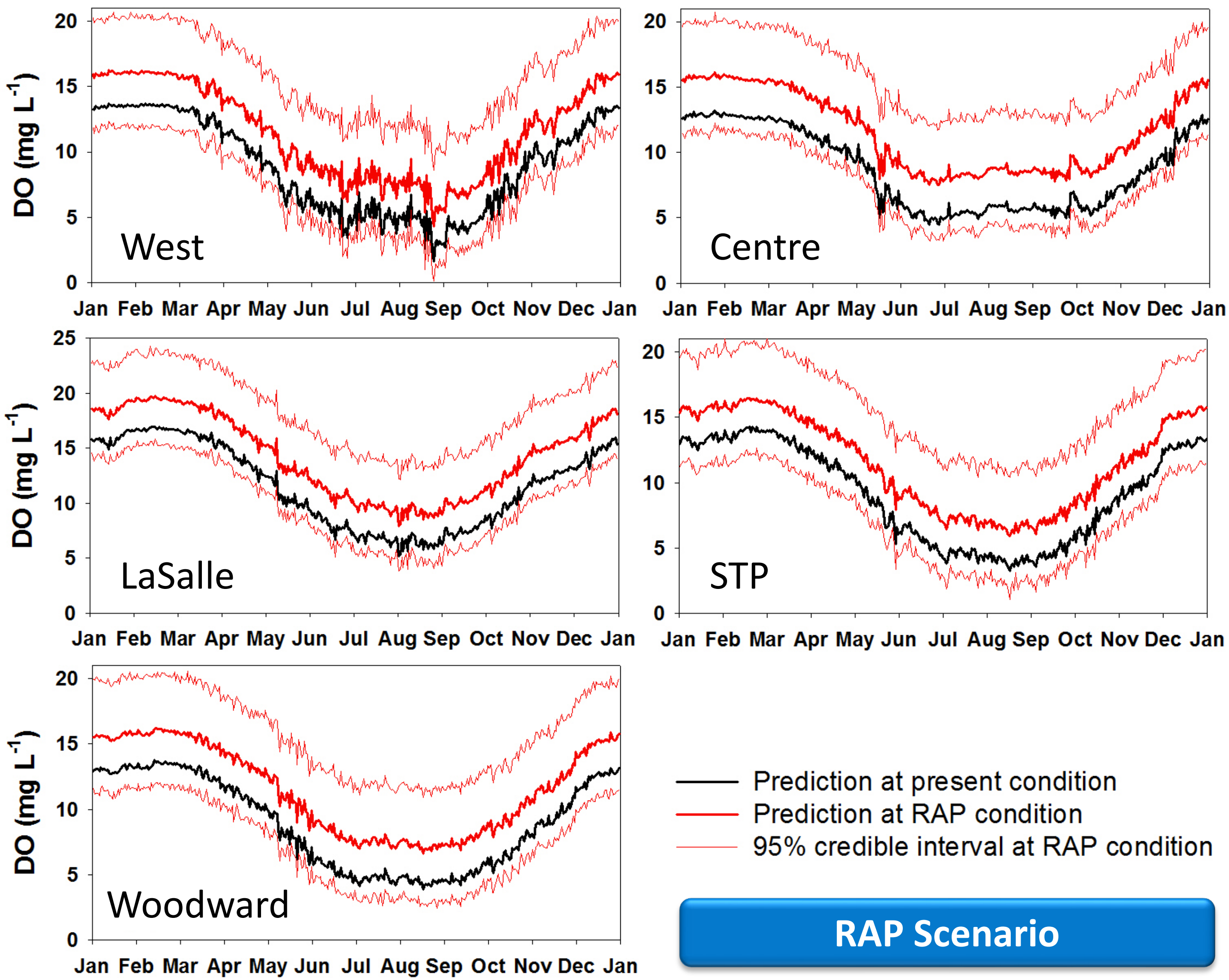
d_{xy} represents the distance between the different pairs of grid cells

ϕ controls the rate of decline of correlation with distance

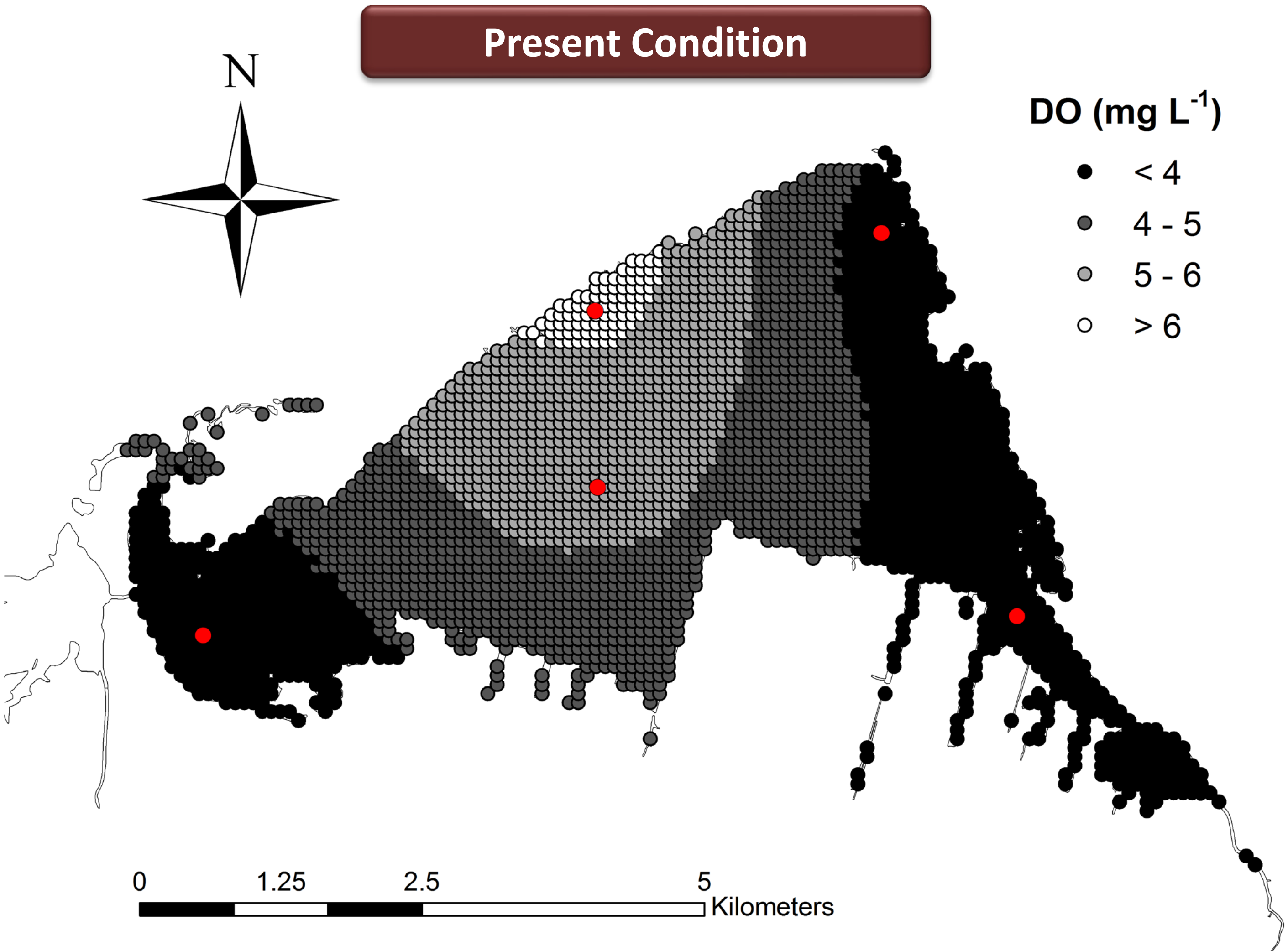
ϕ large –> rapid decay
ϕ small –> slow decay

κ controls the amount by which spatial variations in the data are smoothed.

DO prediction



DO prediction (Jun. to Sep.)



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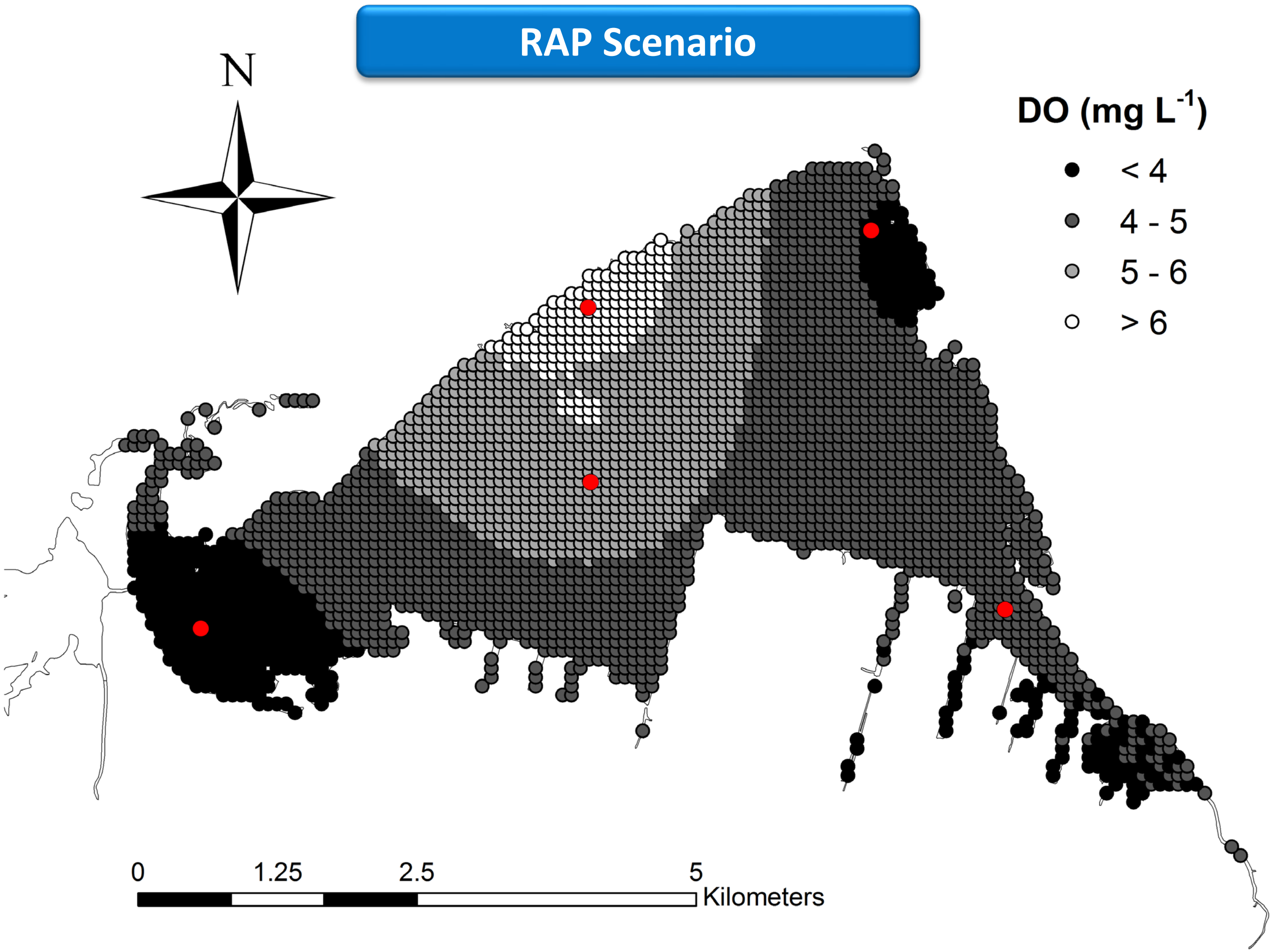
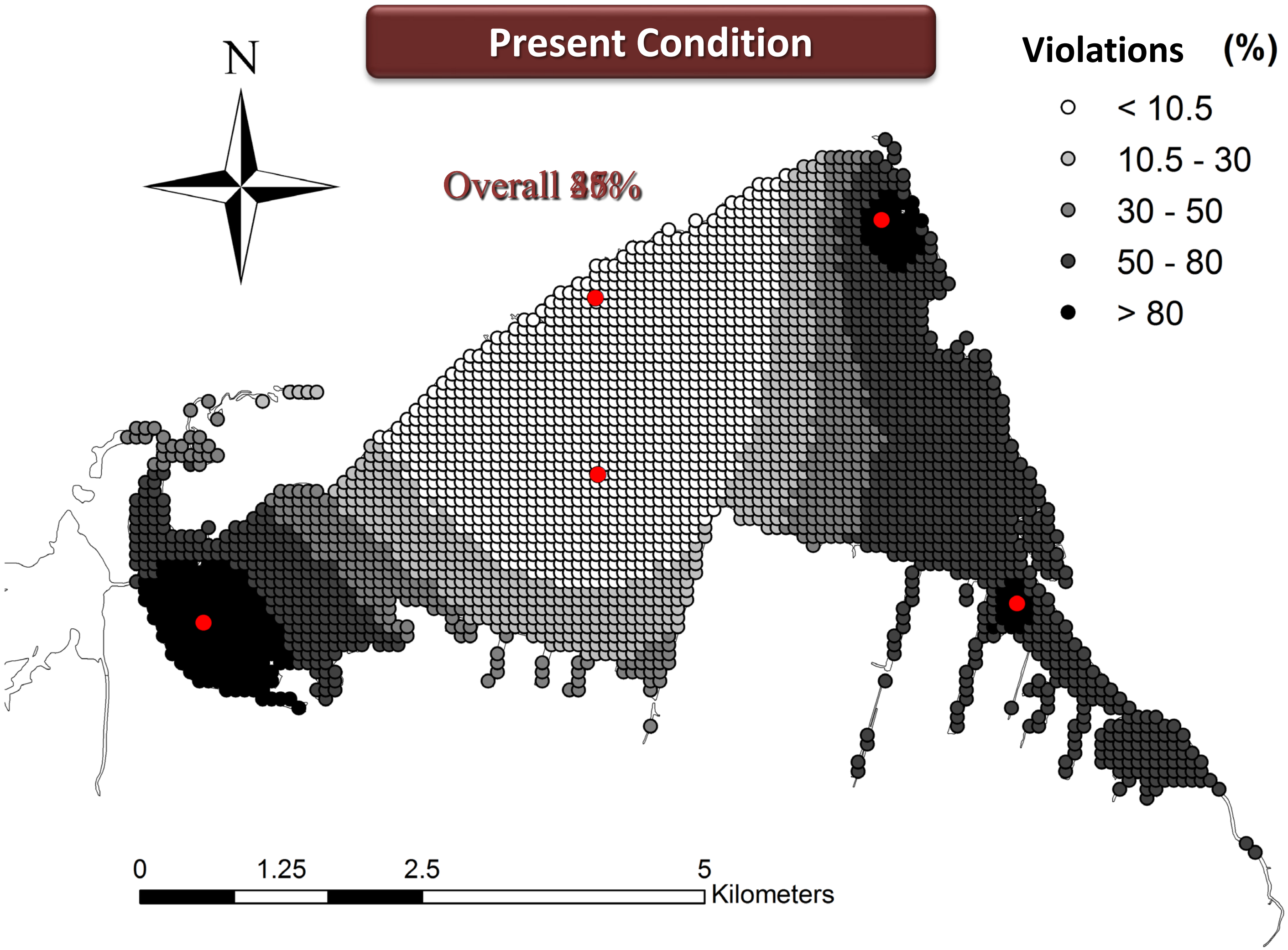


DO violations (<4 mg L^{-1} Jun. to Sep.)



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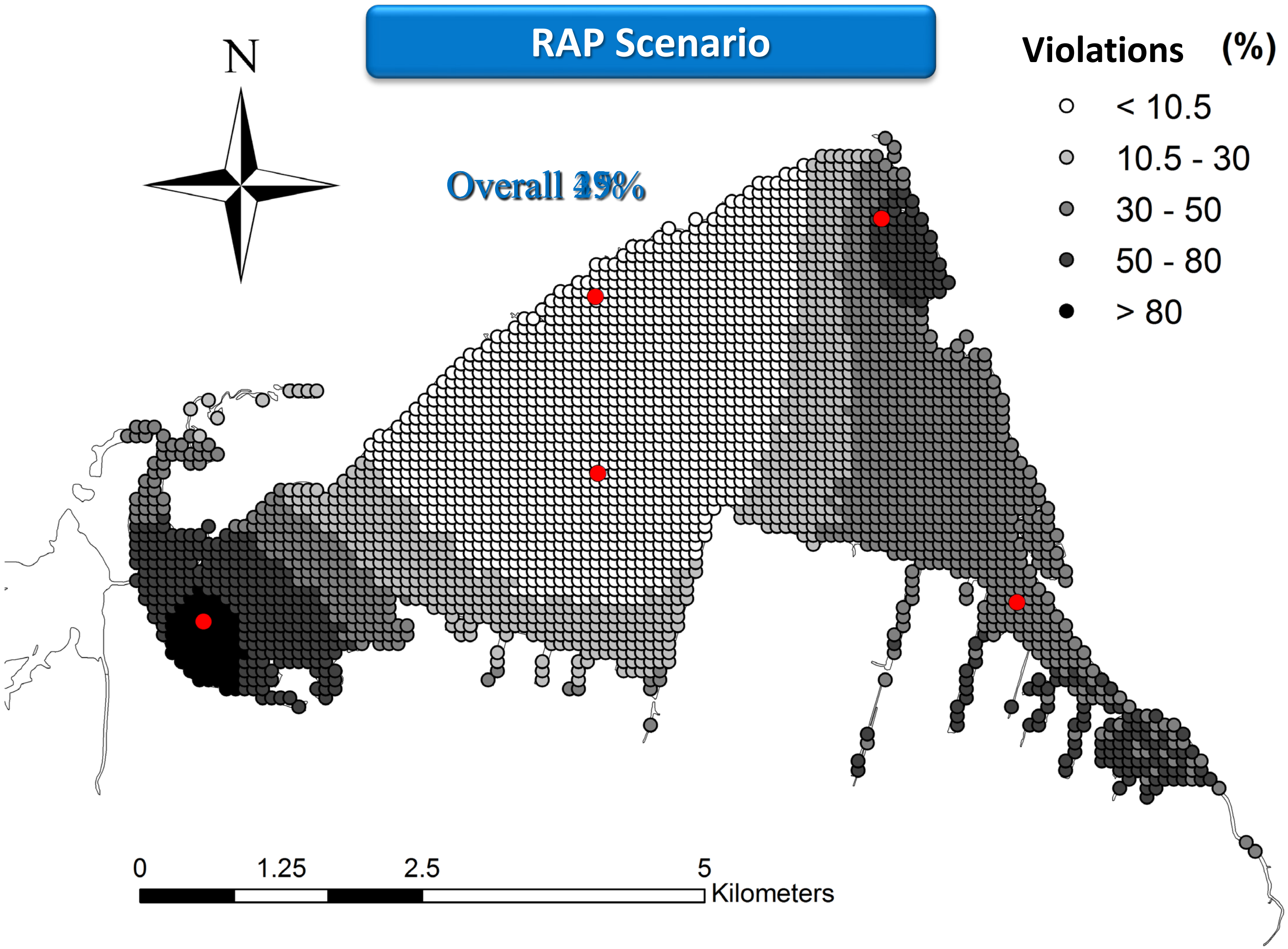

Advantages of our Bayesian Emulator

- **Flexible structure with low computational demands (1/200 of the typical computational time of 3-D hydrodynamic models);**
- **Explicit consideration of all the sources of uncertainty (structural, parametric, natural variability);**
- **Methodological tool that can be augmented by increasing the fidelity of the hydrodynamic component;**
- **Ability to sequentially update beliefs as new knowledge is available, and the consistency with the scientific process of progressive learning and the policy practice of adaptive management.**

Acknowledgements

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MINISTRY OF THE ENVIRONMENT

Fisheries and Oceans Canada

Pêches et Océans Canada

Environment Canada